Defects and Dualities

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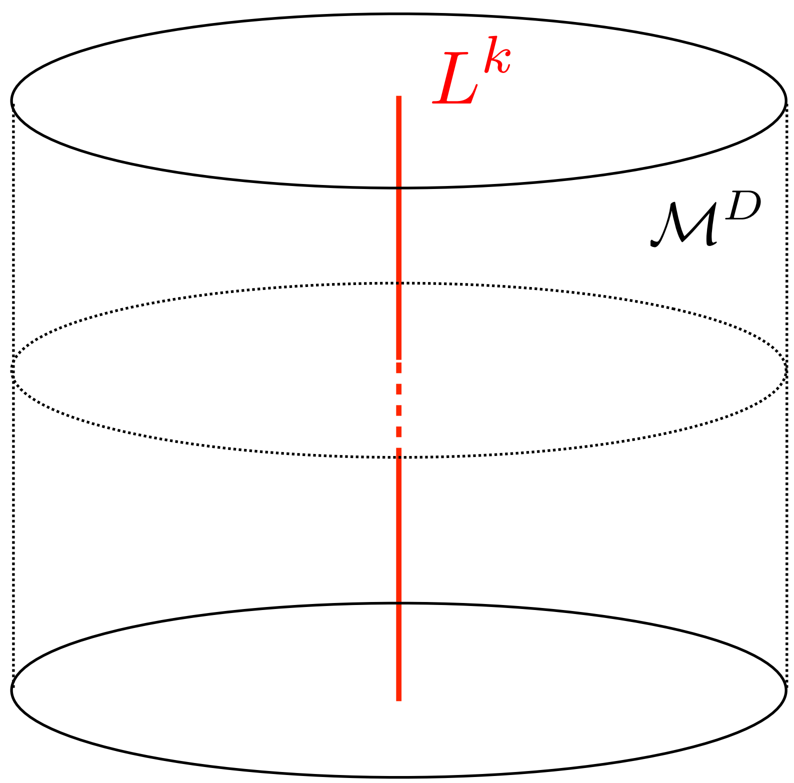
Based on

arXiv:1412.2781 with Davide Gaiotto (Perimeter)

arXiv:1412.6081 with Mathew Bullimore (IAS), Peter Koroteev (Perimeter)

What is a defect ?

A singularity supporting a submanifold $L^k \subset \mathcal{M}^D \quad (D > k)$ in spacetime manifold $\mathcal{M}^D$.

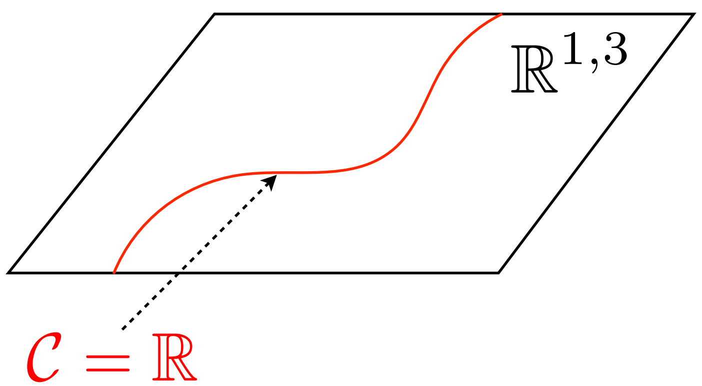


(Codimension $D - k$ defect)

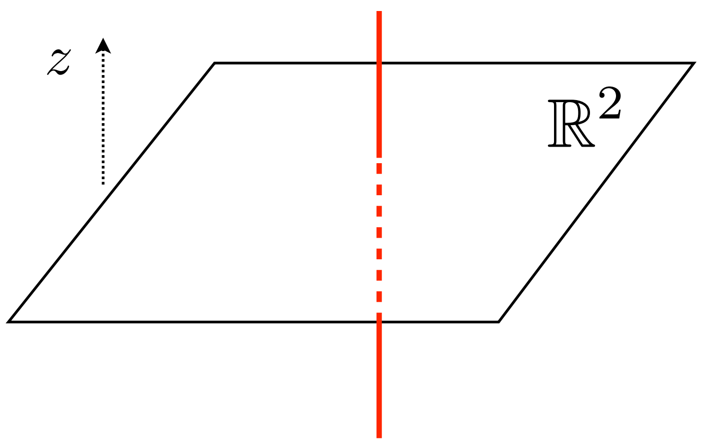
Introduction

What types of defects appear in physics ?

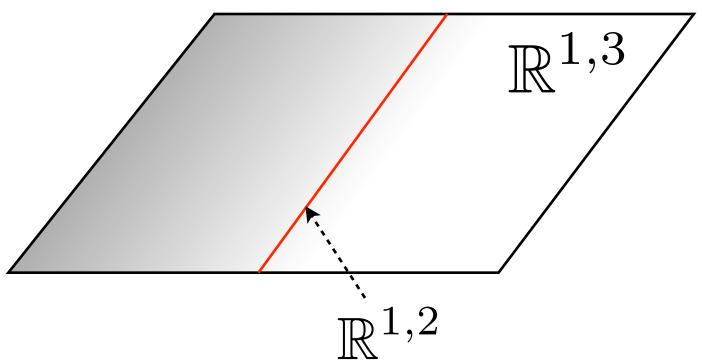
Wilson (or ’t Hooft) line



heavy vortex, cosmic string



Domain wall

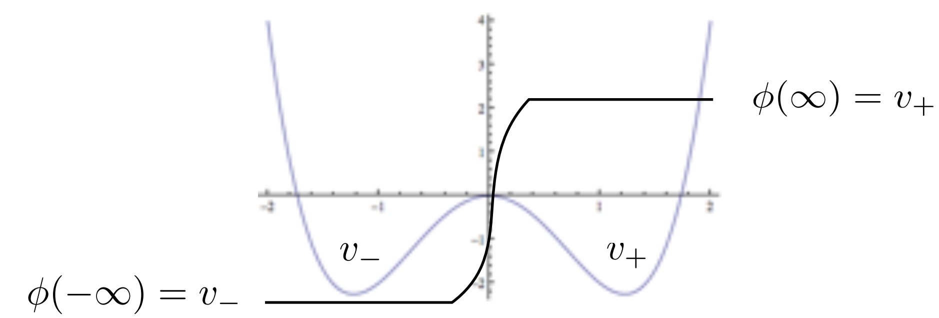


. . . (many other defects)

Introduction

How are defects defined ?

1. Topological defect



2. Singular behavior of fields

$$F_{\mu\nu} dx^\mu \wedge dx^\nu \sim \frac{m}{4} \epsilon_{ijk} \frac{x^i}{|x|^3} dx^j \wedge dx^k$$

3. Orbifold on spacetime

$$\mathcal{M}^D \to \mathcal{M}^D / \mathbb{Z}_N$$

4. Coupling to defect theory

$$S_D[\Psi_D] + S_k[\psi_k] + S_{D,k}[\Psi_D, \psi_k]$$

5. RG flow in Higgs branch

$$\langle B(x,y) \rangle \sim (x+iy)^r$$

Introduction

Why are defects important ?

- They serves as order parameters in phase transition.

  Symmetry breaking, confining phase, Higgs phase, …

- They can observe symmetries or dualities.

  Global structure of symmetries, electric-magnetic duality, mirror duality, …

- They can provide a zoo of theories in lower dimensions after compactification.

  Class S theories, Class R theories, AGT correspondence, …

- so on ….

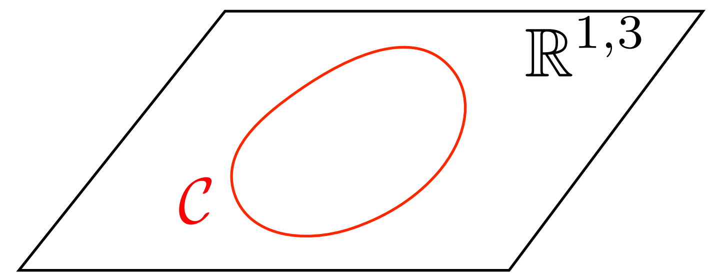
Contents

- Introduction
- Line defects and surface defects in gauge theory
- Defects and duality in 5d gauge theory
- Conclusion

Defects in gauge theory

Line defects

Wilson loop



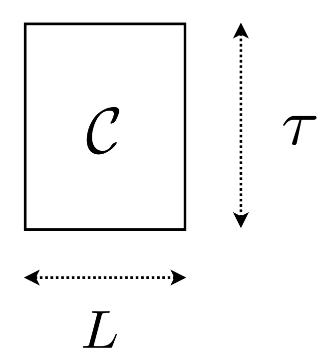
$$W_R = \mathrm{Tr}_R \mathcal{P} \exp\left(i \oint_{\mathcal{C}} A_\mu dx^\mu \right)$$

R : Rep. of gauge group $\mathcal{G}$

- Heavy electrically charged particle

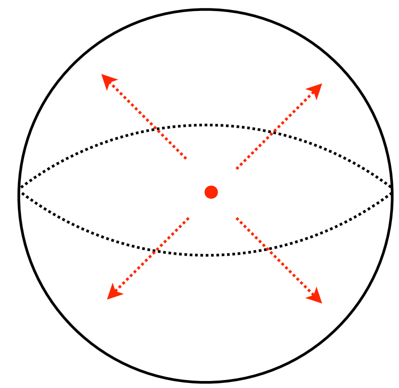
- In confining phase, it exhibits area law : being an order parameter of confining phase

$$\langle W \rangle \sim e^{-\tau V(L)} \ , \ V(L) \sim L$$



Line defects

't Hooft loop (in 4d)



- Disorder operator : defined by singularity

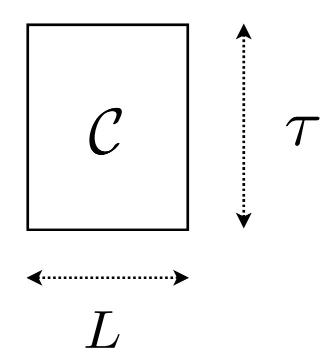
$$F_{\mu\nu} dx^\mu \wedge dx^\nu \sim \frac{m}{4} \epsilon_{ijk} \frac{x^i}{|x|^3} dx^j \wedge dx^k$$

m : magetic charge

- Heavy magnetic monopole

- In Higgs phase, it exhibits area law.

$$\langle T \rangle \sim e^{-\tau V(L)} \ , \ V(L) \sim L$$



Duality on Line defects

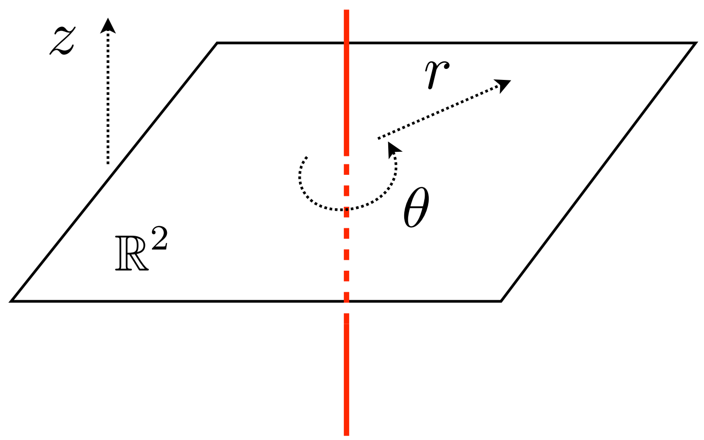
Electric-magnetic duality

$$(\vec{E}, \vec{B}) \to (\vec{B}, -\vec{E}) \ , \ e \to g = -4\pi/e$$

Wilson loop $\longleftrightarrow$ 't Hooft loop

- Duality in 4d gauge theories
- Strong-Weak duality, so hard to check
- Some checks have been done with supersymmetric partition functions

Codimension 2 defects



(Surface defects in 4 dimensions)

1. Singularity of fields as $r \to 0$ (Gukov-Witten defect)

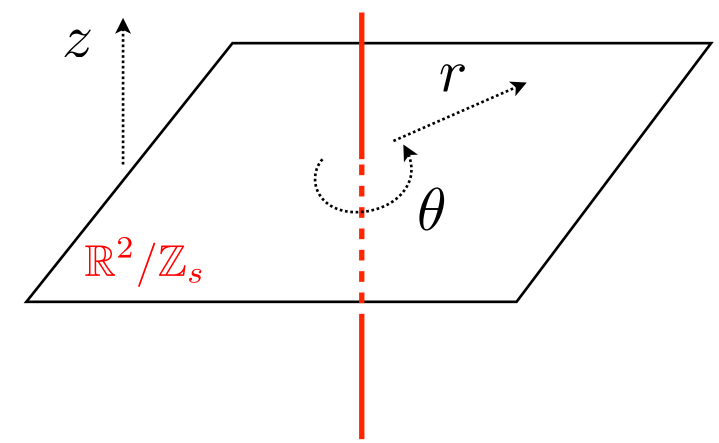
$$A_\mu dx^\mu \sim \mathrm{diag}(\underbrace{m_1, \cdots, m_1}_{n_1}, \underbrace{m_2, \cdots, m_2}_{n_2}, \cdots, \underbrace{m_s \cdots, m_s}_{n_s}) d\theta$$

m_i : monodromy parameters

$$\sum_{i=1}^{s} n_i = N$$

- Gauge symmetry (suppose $SU(N)$ gauge group) is broken, near the defect, to Levi subgroup $\mathbb{L} = S\left(U(n_1) \times \cdots \times U(n_s)\right)$.
- Classified by $\mathbb{L}$ or $\rho = [n_1, n_2, \cdots, n_s]$

Codimension 2 defects



2. Geometric singularity on $\mathbb{R}^2$

- Orbifolding : $\theta \sim \theta + \frac{2\pi}{s}$

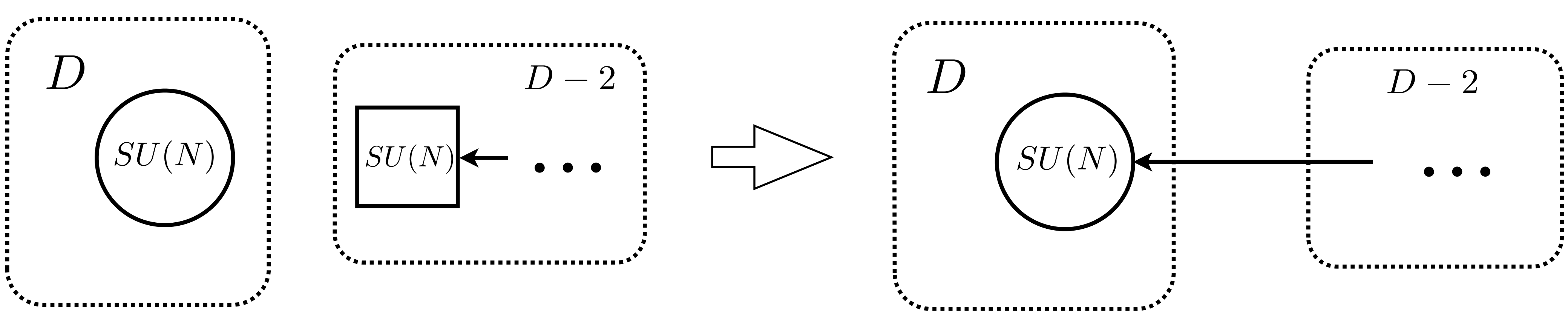
- We can turn on discrete $\mathbb{Z}_s$ holonomies.

$$H = \mathrm{diag}(\underbrace{\omega, \cdots, \omega}_{n_1}, \underbrace{\omega^2, \cdots, \omega^2}_{n_2}, \cdots, \underbrace{\omega^s \cdots, \omega^s}_{n_s}) \qquad (\omega^s = 1)$$

- Gauge symmetry is broken to $\mathbb{L} = S\left(U(n_1) \times \cdots \times U(n_s)\right)$

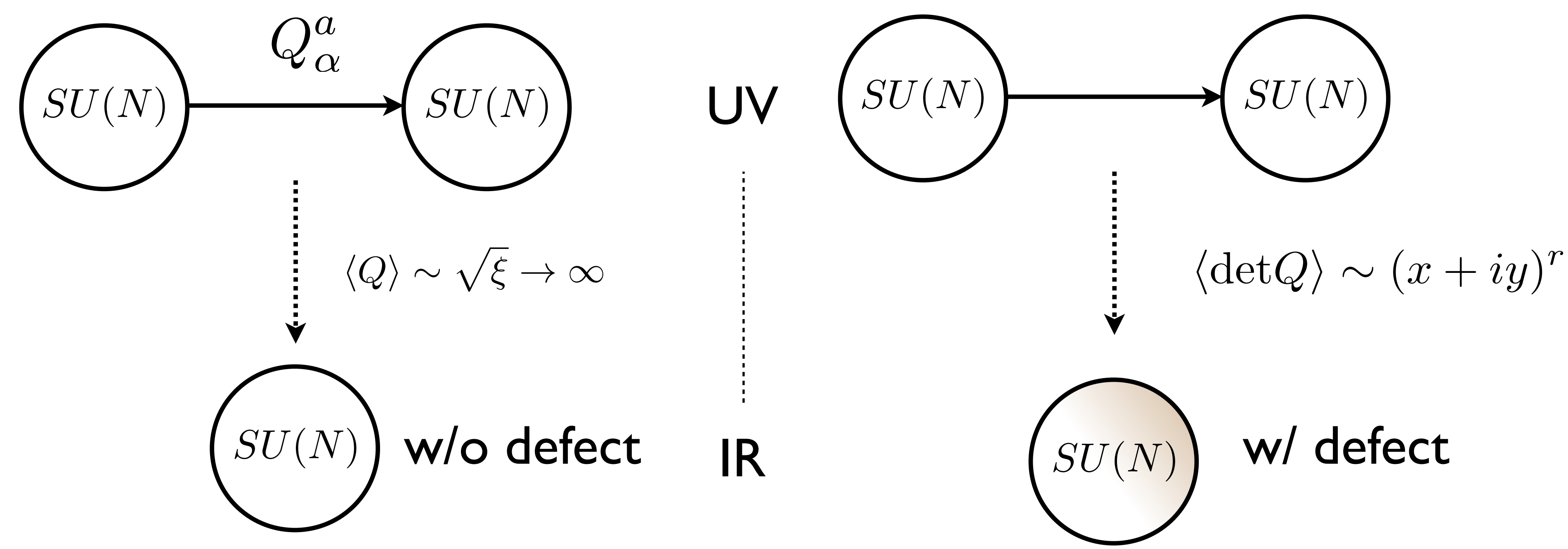
3. Coupling to defect theory

- Consider a $D-2$ dimensional theory (maybe gauge theory) with global symmetry $SU(N)$ and couple it to the bulk $SU(N)$ gauge field.



Codimension 2 defects

4. Higgsing



- Consider a UV $SU(N)\times SU(N)$ theory with a bifundamental matter Q^a_α and move far along Higgs branch.
- We give a large vacuum expectation value, $\langle Q\rangle \sim \sqrt{\xi} \to \infty$.
- Renormalization group flow leads to $SU(N)$ theory in low energy (IR).
- If we instead give a large coordinate dependent vev, $\langle \mathrm{det}Q\rangle \sim (x+iy)^r$, the IR $SU(N)$ theory accommodates codim. 2 defect at $x=y=0$.

Duality on Codimemsion 2 defects

- Mirror symmetry in 3d :

Wilson line $\longleftrightarrow$ Codimension 2 defect

- S-duality in 4d :

Codimension 2 defect $\longleftrightarrow$ Codimension 2 defect

- What about 5d ?

Defect and Duality
in 5d gauge theory

Duality in 5d gauge theory

- 5d gauge theory is non-renormalizable.

- Class of theories admit UV completion as 5d CFT (or 6d CFT), and we are interested in these theories.

- Such theories (at least some of them) have string theory origin and their dualities inherit string theory duality.

ex) Duality in 5-brane web diagram

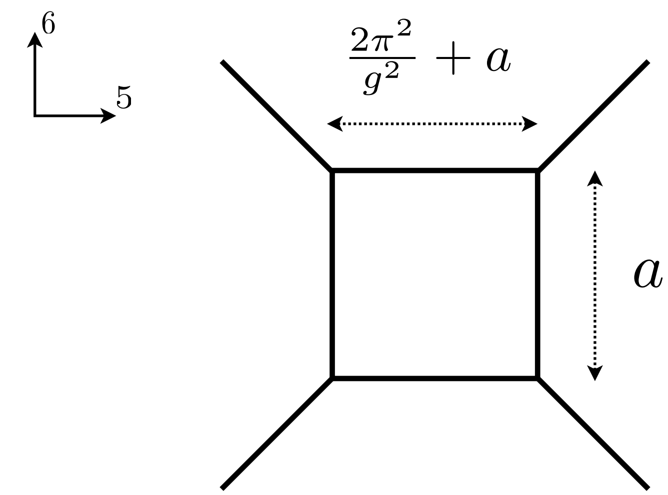
Duality in 5d gauge theory

5d pure $SU(2)$ gauge theory w/o matter field preserving 8 supersymmetries.

5-brane realization

	0	1	2	3	4	5	6	7	8	9
D5	-	-	-	-	-	-				
NS5	-	-	-	-	-		-			

$\underbrace{\qquad\qquad\qquad}_{\text{5d theory}}$ (directions 0–4)



a : Coulomb branch parameter

g^2 : classical gauge coupling

Duality in 5d gauge theory

5d pure $SU(2)$ gauge theory w/o matter field preserving 8 supersymmetries.

5-brane realization

	0	1	2	3	4	5	6	7	8	9
D5	-	-	-	-	-	-				
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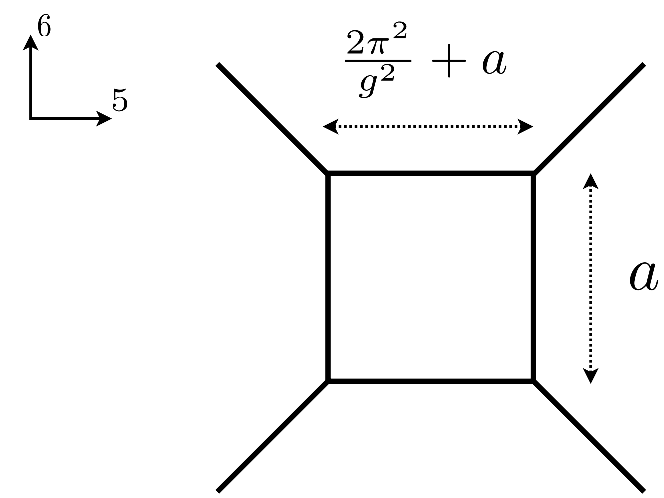
5d theory

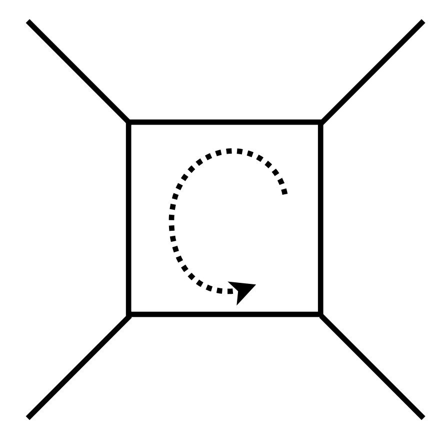


a : Coulomb branch parameter

g^2 : classical gauge coupling

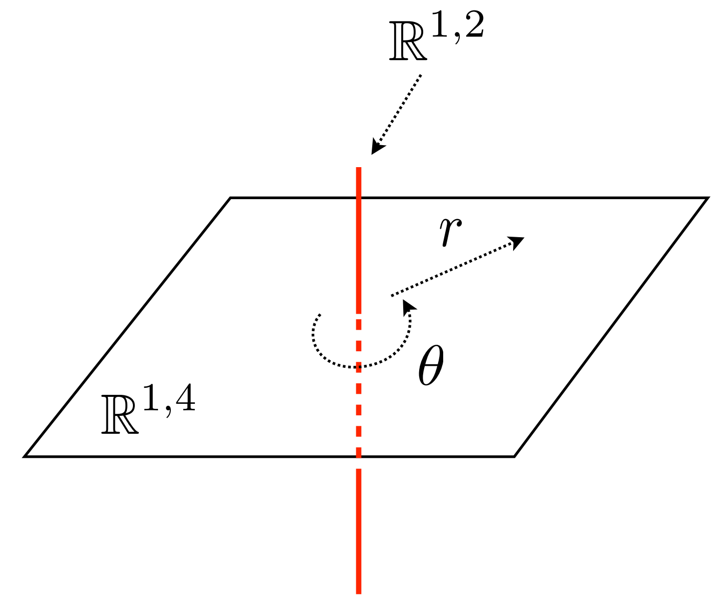
Duality in 5d field theory : $(\tilde{a}, \tilde{g}^2) \rightarrow (\frac{2\pi^2}{g^2} + a, -g^2)$

= 90^o rotation of 5-brane diagram



(It corresponds to $S \subset SL(2,\mathbb{Z})$ action of Type IIB)

Codimension 2 defect of $SU(2)$ gauge theory

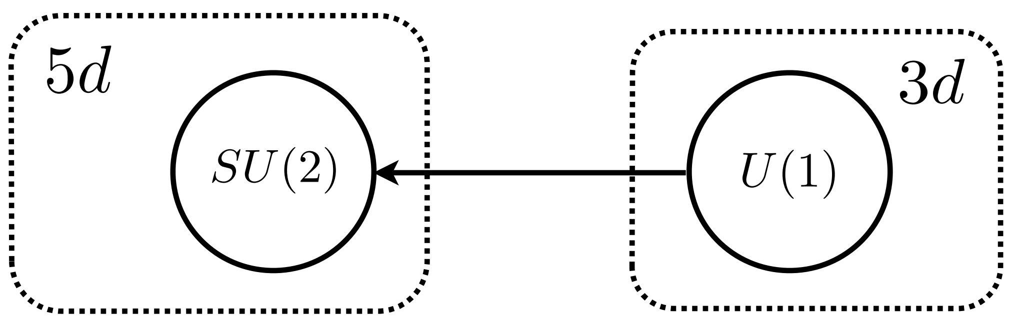


Simplest codimension 2 defect (Type I)

1) Gukov-Witten type : $A_\mu dx^\mu \sim \mathrm{diag}(m, -m) d\theta$

2) $\mathbb{Z}_2$ orbifold : $H = \mathrm{diag}(\omega, \omega^2)$, $\omega^2 = 1$

3) Coupling to 3d $U(1)$ gauge theory with 2 fundamental chiral multiplets



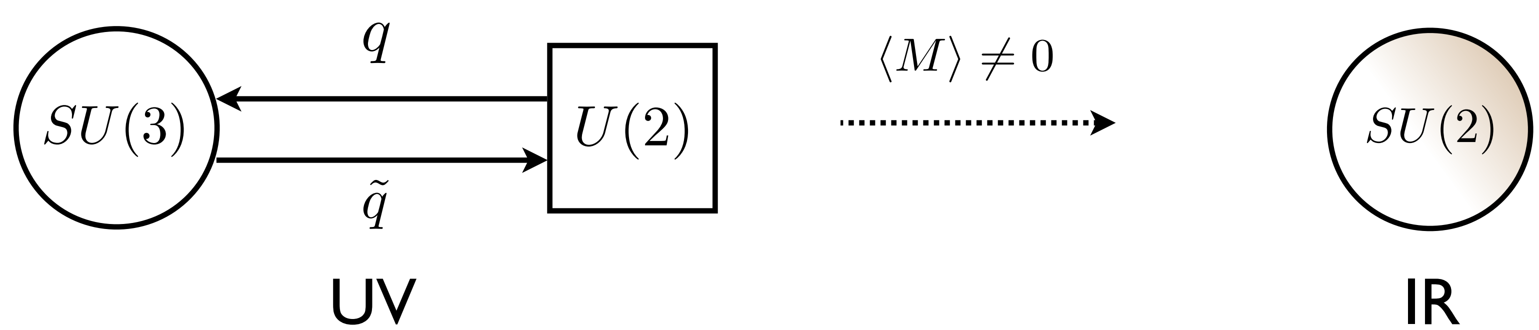
4) Higgsing $SU(2) \times SU(2)$ gauge theory by giving a vev $\langle \mathrm{det} Q \rangle \sim (x + iy)^{r=1}$

All same defects !!

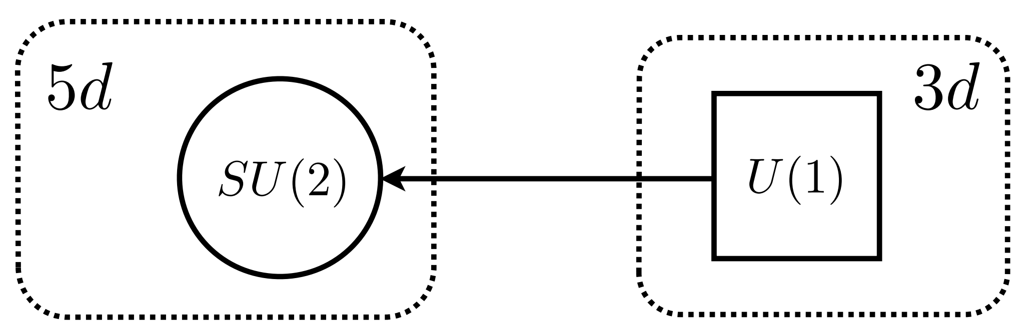
Codimension 2 defect of $SU(2)$ gauge theory

Codimension 2 defect (Type II)

1) Higgsing $SU(3)$ gauge theory with 2 fundamental matters $q, \tilde{q}$ by giving a position dependent vev to mesonic operator $\langle M \rangle = \langle q\tilde{q} \rangle \neq 0$.

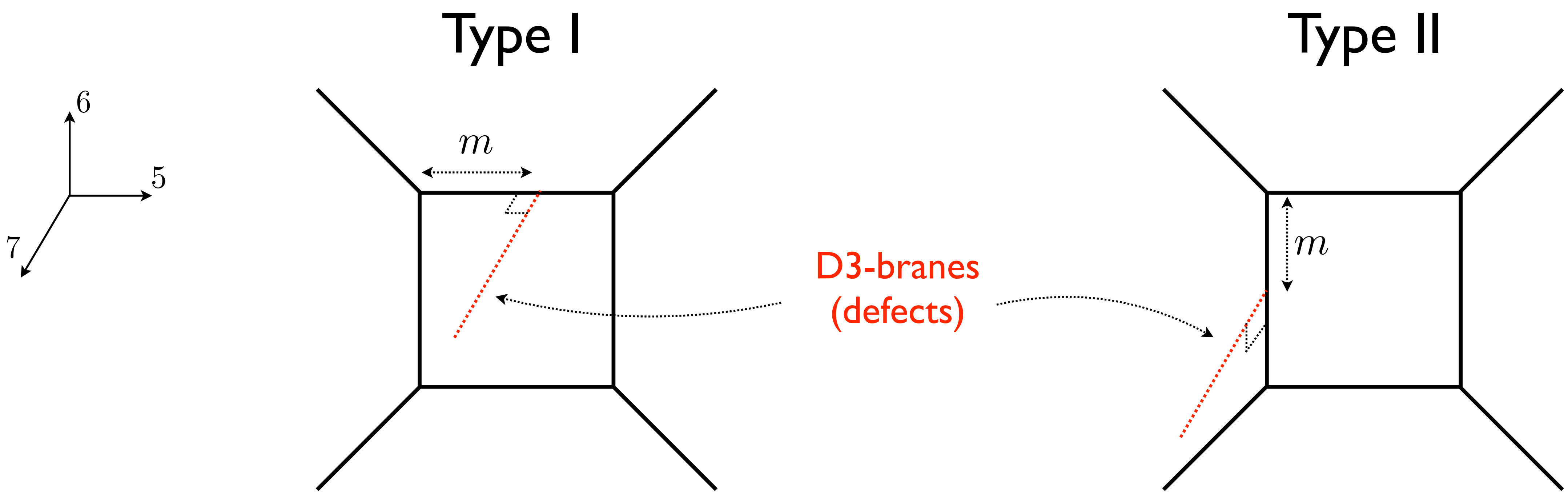


2) Coupling to a couple of 3d free chiral multiplets.



Same defects !!

Brane realization of codimension 2 defects

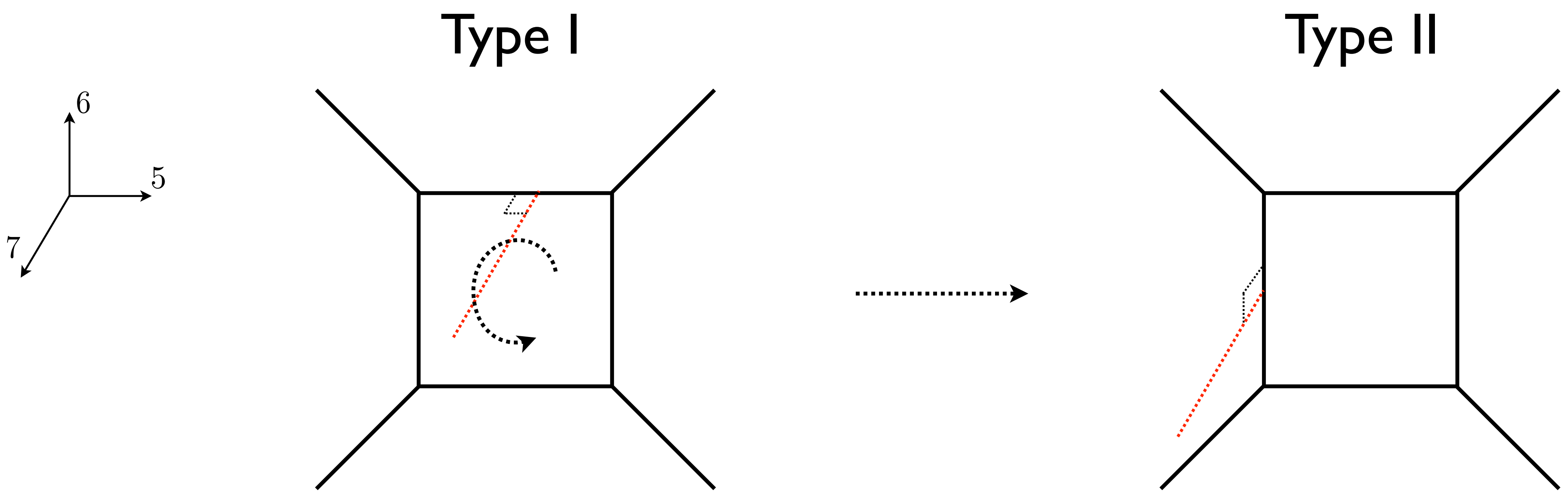


5d theory (over 0 1 2 3 4)

	0	1	2	3	4	5	6	7	8	9
D5	-	-	-	-	-	-				
NS5	-	-	-	-	-		-			
D3	-	-	-					-		

3d theory (over 0 1 2)

Brane realization of codimension 2 defects



Duality in field theory = 90^o rotation of brane-diagram

Defect partition function and Duality test

We have computed defect partition functions (on $S^1 \times \mathbb{R}^4$) and tested duality.

Exact partition functions can be computed using supersymmetric localization.

Supersymmetric localization :

1. Deform path integral in SUSY manner w.r.t supercharge Q.

$$Z^{3d/5d} \quad \rightarrow \quad Z^{3d/5d}(t) = \int \mathcal{D}\Psi e^{-S_E[\Psi] - tQV} \qquad (Q^2 V = 0)$$

2. The result is independent of the deformation

$$\frac{d}{dt} Z^{3d/5d}(t) \sim \int \mathcal{D}\Psi \; Q\Big(e^{-S_E[\Psi] - tQV} V\Big) = 0$$

3. Send $t \rightarrow \infty$ and do gaussian integral around saddle point of $QV = 0$, which gives rise to Exact Result since $Z(t=0) = Z(t=\infty)$.

Defect partition function and Duality test

Type I (GW) $$Z_{(I)} = Z_{(I)}(a, \frac{4\pi^2}{g^2}; m)$$

a : Coulomb branch parameter
g^2 : gauge coupling
m : 3d monodromy parameter

Type II (free chirals) $$Z_{(II)} = Z_{(II)}(\tilde{a}, \frac{4\pi^2}{\tilde{g}^2}; \tilde{m})$$

$\tilde{a}$: Coulomb branch parameter
$\tilde{g}^2$: gauge coupling
$\tilde{m}$: 3d mass parameter

Duality transforms $$(\tilde{a}, \tilde{g}^2) \;\to\; (\frac{2\pi^2}{g^2} + a, -g^2)$$

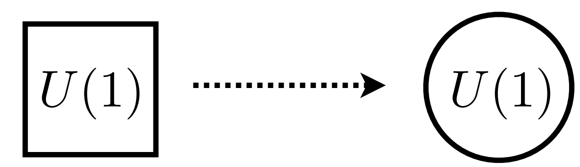
We have checked that two partition functions completely agree.

$$Z_{(I)} = Z_{(II)}$$

up to $(\tilde{a} = \frac{2\pi^2}{g^2} + a, \tilde{g^2} = -g^2, \tilde{m} = m)$

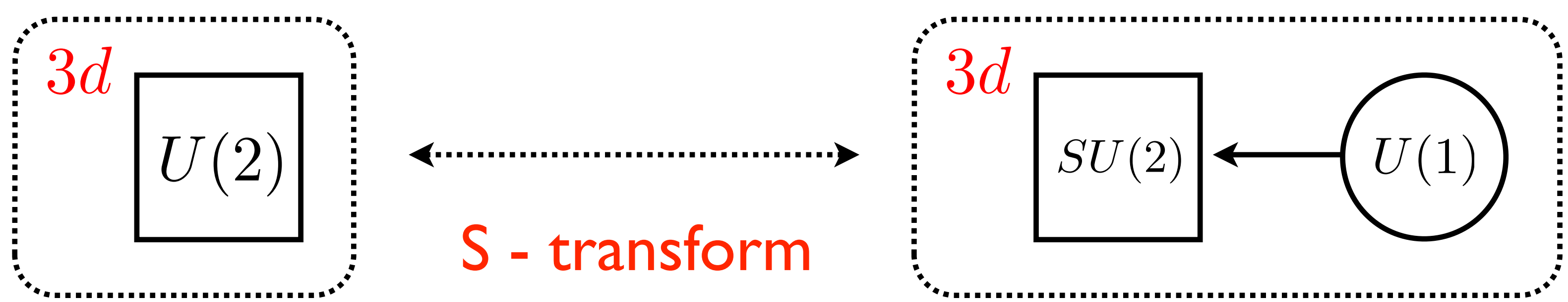
Duality in 3d/5d system

New duality by 3d $S \subset SL(2,\mathbb{Z})$ transformation

- 3d $S \subset SL(2,\mathbb{Z})$ action (not duality in general)
    1. Gauging a $U(1)$ global symmetry :
    

    2. Adding a mixed Chern-Simons term : $\int d^3x\, A_{\rm new} dA_{\rm old}$

In our example



- The same S action on 3d/5d system becomes duality !

$$Z_{(II)}(a, \tfrac{4\pi^2}{g^2}; m) \;\xleftrightarrow{\text{S - transform}}\; Z_{(I)}(a, \tfrac{4\pi^2}{g^2}; m) = Z_{(II)}(\tilde{a}, \tfrac{4\pi^2}{\tilde{g}^2}; \tilde{m})$$

Conclusion

Conclusions

- There are various defects in physics and they play important roles.
- We have studied interesting properties of defects under dualities.

- Combination of different types of defects.

- Dualities on defect theories and those of bulk theories.

- Integrability : $\hat{O}Z = 0$ becomes Baxter equations in integrable systems.

- Implications to realistic system.